

A New Class of Polyphase Zero Correlation Zone Sequence Sets

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Abstract:

In this paper, a novel method for constructing polyphase Zero Correlation Zone (ZCZ) sequence sets based on Discrete Fourier Transform (DFT) matrix and an interleaving technique is proposed. The correlation functions of the proposed sequence set is zero for the phase shifts within the zero correlation zone and their construction is more flexible than other polyphase ZCZ constructions. We show that such a sequence construction can achieve a mathematical upper bound on the ZCZ sequence sets.

Keywords: Polyphase sequences, mathematical upper bound, Zero Correlation Zone (ZCZ) sequences, correlation, DFT matrix.

1. Introduction

Code Division Multiple Access (CDMA) is a popular channel access method that allows high-speed communications (Driz, 2018; Fassi, 2014). CDMA technique is employed for various applications notably the radio communication such as 4G Long Term Evolution (LTE) and 5G New Radio (NR) systems, in which the spreading sequences are designed to ensure very low Auto-Correlation and very low Cross-Correlation Functions (ACF and CCF) at out-of-phase state (Sun et al., 2017; Sarkar and Majhi, 2020). This means that distortion forms of the signal such as Inter Symbol Interference (ISI) and Multiple Access Interference (MAI), can be reduced by using a good codes family having good correlation properties (Driz and Djebbari, 2019; Fassi and Taleb-Ahmed, 2018a).

Recently, an interesting class of spreading sequences called Zero Correlation Zone (ZCZ) is proposed as being an innovative spreading that achieves a new concept to CDMA system scheme (Fassi and Taleb-Ahmed, 2018b; Ghali et al., 2021). Any $ZCZ(L, M, Z_{cz})$ sequence sets characterized by the sequence length L , the number of sequences M and the length of the zero-correlation zone Z_{cz} , ensures ideal autocorrelation and cross-correlation properties within the correlation zone Z_{cz} (Conti, 2006; Zhou et al., 2017; Driz et al., 2019; Pai and Chen, 2021). In the other hand, a ZCZ sequence set that satisfies the mathematical upper bound

defined by the ratio $M(Z_{cz} + 1)/L = 1$ is called an optimal Zero-Correlation Zone sequence set (Torii et al., 2005).

Diverse ZCZ code constructions have been reported in literature the of which we can cite binary, ternary, polyphase and optical codes (Ouis et al., 2020; Maeda et al., 2010; Fassi et al., 2013; Fassi et al., 2014; Chen et al., 2021; Kuroda et al., 2020; Zhang et al., 2020; Feng et al., 2015).

This paper proposes a new class of polyphase ZCZ sequence sets. Compared with other existing works on polyphase ZCZ sequence sets, the proposed ZCZ sequence set is simple and more flexible. The remainder of the paper is organized into six sections. After a review of preliminary considerations in Section 2, the proposed sequence construction is described in detail in Section 3. Section 4 gives examples of the novel ZCZ sequence sets. The properties of the proposed sequence sets are described in Section 5 and finally concluding remarks are reported in the conclusion Section.

2. Preliminaries

2.1 Definition 1

The Periodic Correlation Function between $X_s = (x_{s,0}, x_{s,1}, \dots, x_{s,L-1})$ and, $X_v = (x_{v,0}, x_{v,1}, \dots, x_{v,L-1})$ at a shift τ is defined by (Fassi et al., 2013) :

$$\forall \tau \geq 0, \theta_{(X_s, X_v)}(\tau) = \sum_{i=0}^{L-1} x_{s,i} x_{v,(i+\tau) \bmod L} \text{ and } \theta_{(X_s, X_v)}(-\tau) = \theta_{(X_v, X_s)}(\tau) \quad (1)$$

2.2 Definition 2

A set of M sequences $\{X_0, X_1, X_2, \dots, X_{M-1}\}$ is denoted by $\{X_s\}_{s=0}^{M-1}$.

A set of sequences $\{X_s\}_{s=0}^{M-1}$ is called ZCZ sequence set, denoted by $ZCZ(L, M, Z_{cz})$ if the periodic correlation function satisfy (Maeda et al., 2010) :

$$\forall s, 0 < |\tau| \leq Z_{cz}, \theta_{(X_s, X_s)}(\tau) = 0$$

(2)

$$\forall j, (s \neq v), |\tau| \leq Z_{cz}, \theta_{(X_s, X_v)}(\tau) = 0$$

(3)

3. Proposed Sequence Construction

In this section, a new method for constructing polyphase ZCZ sequence sets based on DFT matrix is proposed. The construction is accomplished through three steps.

Step 1

Firstly, let F be the N -point DFT matrix as:

$$F = \begin{bmatrix} W_N^0 & W_N^0 & \dots \\ W_N^0 & W_N^1 & \dots \\ \vdots & \vdots & W_N^{(N-1)(N-1)} \end{bmatrix} \quad (4)$$

Where $W_N = \exp(-2\pi j/N)$ and $N > 2$.

The s^{th} row of the matrix F of order N is denoted by $f_s = [f_{s,0}, f_{s,1}, \dots, f_{s,N-1}]$. A set of $2N$ sequences p_s , each of length $2N$, is constructed as follows (Fassi et al., 2013):

For $0 \leq s < N$,

$$p_{s+0} = [f_s, -f_s] \quad (5)$$

$$p_{s+1} = [f_s, f_s] \quad (6)$$

Step 2

For a fixed integer value $N > 2$, and for the first stage, $i = 0$, we can generate, based on the scheme for sequence construction in (Maeda et al., 2010), a series of sets $\{P_s\}_{s=0}^{2N-1}$ of $2N$ sequences as follows:

- A sequence set $\{P_s\}_{s=0}^{2N-1}$ is constructed from the sequences set $\{p_s\}_{s=0}^{2N-1}$.
- A pair of sequences P_{s+0} and P_{s+1} of length $(2^{i+2}N)$ are constructed by the process of interleaving a sequence pair p_{s+0} and p_{s+1} , as follows:

For $0 \leq s < N$,

$$P_{s+0} = [p_{s+0,0}, p_{s+1,0}, p_{s+0,1}, p_{s+1,1}, \dots, p_{s+0,2N-1}, p_{s+1,2N-1}] \quad (7) \quad \text{and,}$$

$$P_{s+1} = [p_{s+0,0}, -p_{s+1,0}, p_{s+0,1}, -p_{s+1,1}, \dots, p_{s+0,2N-1}, -p_{s+1,2N-1}] \quad (8)$$

The member size of the sequence set $\{P_s\}$ is $2N$.

Step 3

For $i > 0$, we can recursively construct a new series of set, $\{P_s\}_{s=0}^{2N-1}$, by interleaving of actual $\{P_s\}_{s=0}^{2N-1}$. The $\{P_s\}_{s=0}^{2N-1}$ is generated as follows:

For $0 \leq s < N$,

$$P_{s+0} = [P_{s+0,0}, P_{s+1,0}, P_{s+0,1}, P_{s+1,1}, \dots, P_{s+0,4N-1}, P_{s+1,4N-1}] \quad (9)$$

and,

$$P_{s+1} = [P_{s+0,0}, -P_{s+1,0}, P_{s+0,1}, -P_{s+1,1}, \dots, P_{s+0,4N-1}, -P_{s+1,4N-1}] \quad (10)$$

The length of both sequences P_{s+0} and P_{s+1} is equal to $(2^{i+2}N)$.

4. Examples of Construction

4.1 Example 1

Let F be a matrix of order $N = 4$, given by:

For $i = 2$, and from Equations (9) and (10), the sequence elements of $\{P_s\}_{s=0}^7$ set with $ZCZ(L, M, Z_{cz}) = (64, 8, 4)$ is given below, where the symbols 0, 1, 2, 3 represent respectively $+1, +j, -1, -j$ and $+j = \sqrt{-1}$.

P_{3+1} 0,2,0,0,0,2,2,2,3,1,3,3,3,1,1,1,2,0,2,2,2,0,0,0,1,3,1,1,1,3,3,3,
2,0,2,2,0,2,2,2,1,3,1,1,3,1,1,1,0,2,0,0,2,0,0,0,3,1,3,3,1,3,3,3

$$\begin{aligned} & |\theta_{(P_{0+1}, P_{1+1})}(\tau)| \\ &= (0, 0, 0, 0, 0, 4, 0, 4, 16, 4, 0, 4, 0, 0, 0, 0, 0, 0, 0, 4, 0, 4, 16, 4, 0, 4, 0, 0, 0, 0, 0, 0, 0, 4, 0, 4, \\ & \quad , 16, 4, 0, 4, 0, 0, 0, 0, 0, 0, 0, 0, 4, 0, 4, 16, 4, 0, 4, 0, 0, 0, 0) \end{aligned}$$

Fig 1 shows the auto-correlation function ($\forall s, s = v$) given in Equation (1) of P_{0+1} , and Fig 2 shows the cross-correlation function ($\forall s, s \neq v$) given in Equation (1) of P_{0+1} with P_{1+1} .

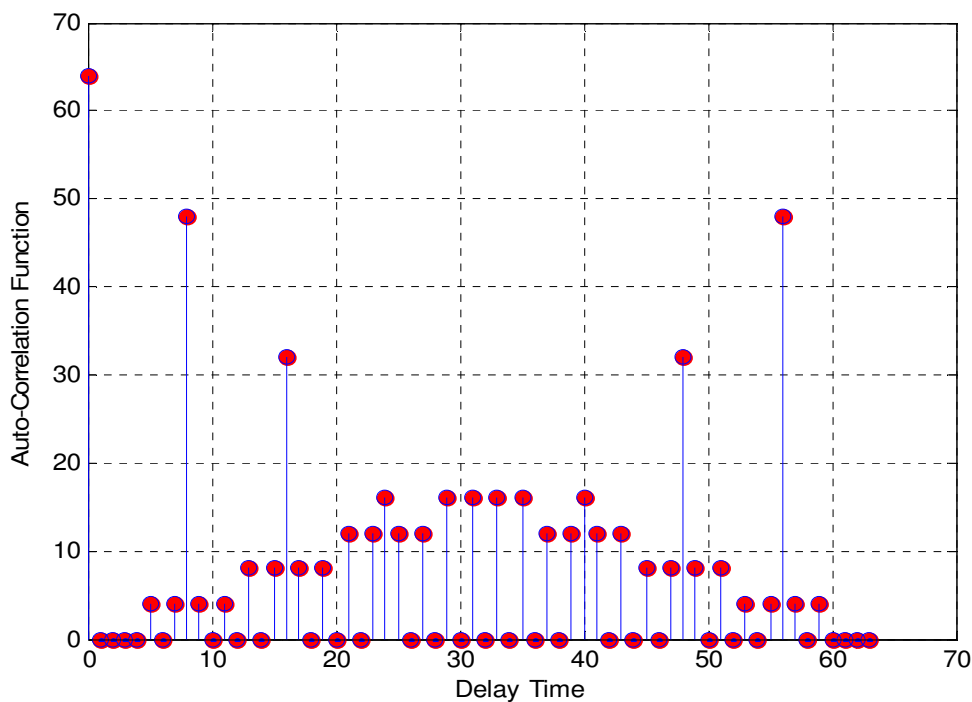


Fig 1. The ACF of P_{0+1} with ZCZ(64,8,4)

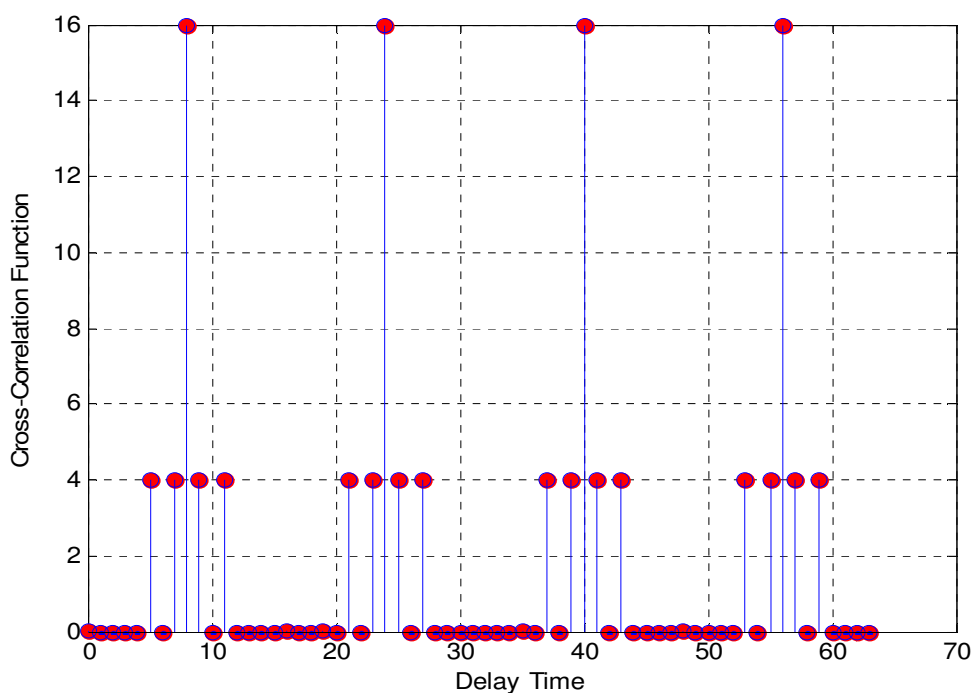


Fig 2. The CCF between P_{0+1} and P_{1+1} with ZCZ(64,8,4)

The correlation functions confirm that $\{P_j\}$ is a ZCZ (64,8,4) sequence set.

4.2 Example 2

Another example is given for $N = 3$, let F be :

$$F = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \exp(-2\pi j/3) & \exp(-4\pi j/3) \\ 1 & \exp(-4\pi j/3) & \exp(-2\pi j/3) \end{bmatrix}$$

For $i = 1$, and from the Equations (9) and (10), the sequence elements of the proposed $\{P_s\}_{s=0}^5$ set with $ZCZ(L, M, Z_{cz}) = (24, 6, 2)$ is given below, where the symbols 0, 1, 2, 3, 4, 5, represent respectively $+1, \exp(-\frac{2\pi j}{3}), -1, \exp(-\frac{4\pi j}{3}), -\exp(-\frac{2\pi j}{3})$ and $-\exp(-\frac{4\pi j}{3})$.

$$P_{0+0} \quad 0,0,0,2,0,0,0,2,0,0,0,2,2,2,0,2,2,2,0,2,2,2,0,2$$

$$P_{0+1} \quad 0,0,0,2,3,3,3,5,1,1,1,4,2,2,0,2,5,5,3,5,4,4,1,4$$

$$P_{1+0} \quad 0,0,0,2,1,1,1,4,3,3,3,5,2,2,0,2,4,4,1,4,5,5,3,5$$

$$P_{1+1} \quad 0,2,0,0,0,2,0,0,0,2,0,0,2,0,0,2,0,0,0,2,0,0,0,0$$

$$P_{2+0} \quad 0,2,0,0,3,5,3,3,1,4,1,1,2,0,0,0,5,3,3,3,4,1,1,1$$

$$P_{2+1} \quad 0,2,0,0,1,4,1,1,3,5,3,3,2,0,0,0,4,1,1,1,5,3,3,3$$

The autocorrelation function of P_{0+1} is given by:

$$|\theta_{(P_{0+1}, P_{0+1})}(\tau)| = (24, 0, 0, 4, 16, 4, 0, 8, 8, 8, 0, 12, 0, 12, 0, 8, 8, 8, 0, 4, 16, 4, 0, 0)$$

The cross-correlation function between P_{0+1} and P_{1+1} is also given by:

$$|\theta_{(P_{0+1}, P_{1+1})}(\tau)| = (0, 0, 0, 4, 0, 4, 0, 4, 0, 4, 0, 0, 0, 0, 4, 0, 4, 0, 4, 0, 4, 0, 0)$$

Fig 3 shows the auto-correlation function of P_{0+1} , and Fig 4 shows the Cross-Correlation Function of P_{0+1} with P_{1+1} .

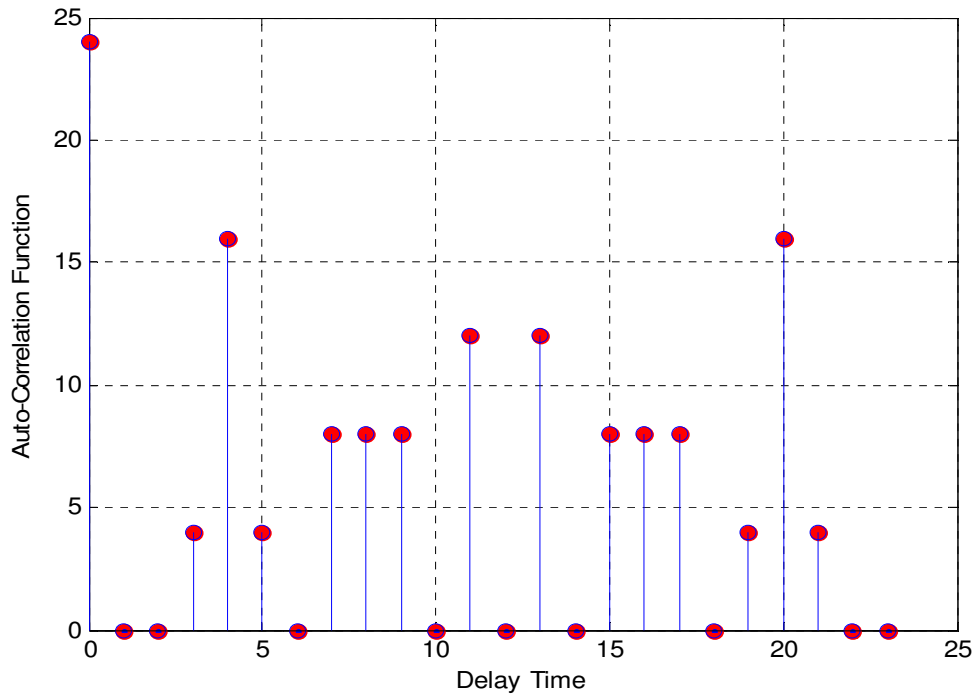


Fig 3.The ACF of P_{0+1} with ZCZ(24,6,2)

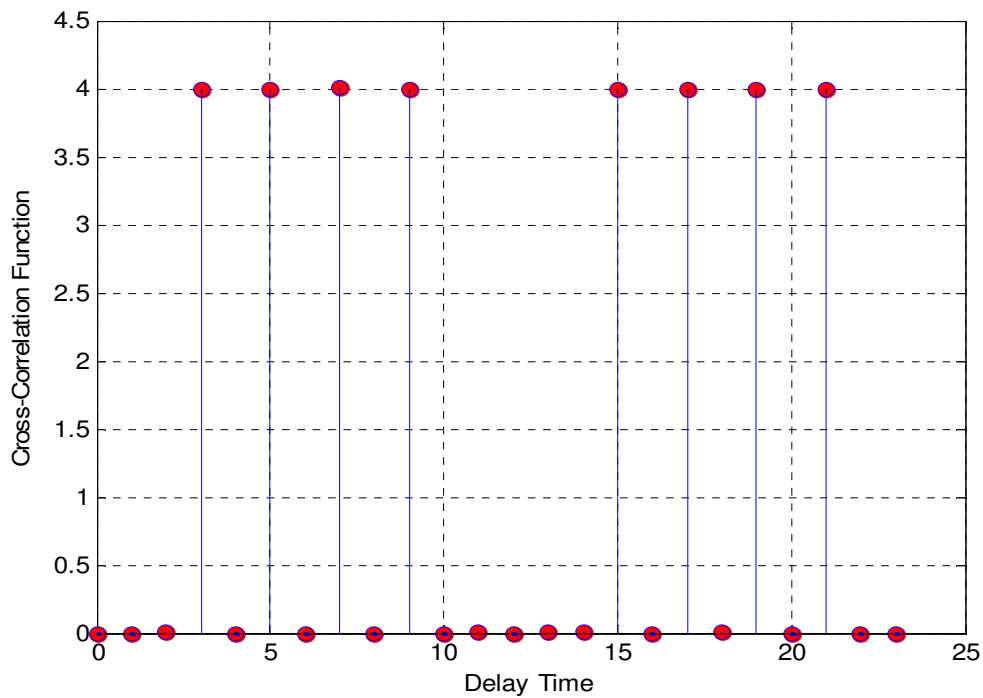


Fig 4.The CCF between P_{0+1} and P_{1+1} with ZCZ(24,6,2)

The correlation functions confirm that $\{P_j\}$ is a ZCZ (24,6,2) sequence set.

5. Features of the Proposed Sequence

The proposed polyphase ZCZ sequence set can be generated from a DFT matrix of order $N > 2$ and their construction is more flexible than other polyphase ZCZ constructions (Suehiro, 1994; Torii et al., 2005; Sudha Rani R. and Vittal Reddy, 2011).

The obtained polyphase ZCZ sequence set has two binary sequences for the index row $M = 1$ and $M = N + 1$. This property came from elements of the DFT matrix, $[W_N^{M-1} = \exp(-2\pi j(M-1)/N)]$. The generated polyphase ZCZ sequence set satisfies the following properties (Maeda et al., 2010):

$$\begin{cases} \forall s, \forall \tau \neq 0, |\tau| \leq 2^i \\ \theta_{(P_s, P_s)}(\tau) = 0 \end{cases} \quad (11)$$

and,

$$\begin{cases} \forall s \neq v, \forall \tau, |\tau| \leq 2^i \\ \theta_{(P_s, P_v)}(\tau) = 0. \end{cases} \quad (12)$$

The proposed polyphase ZCZ sequence set with $ZCZ(L, M, Z_{CZ}) = (2^{i+2}N, 2N, 2^i)$ is optimal or approach optimal polyphase ZCZ sequences.

Proof

For the proposed $ZCZ(2^{i+2}N, 2N, 2^i)$ sequence set, $\rho = \frac{M(Z_{CZ}+1)}{L} = \frac{2N(2^i+1)}{2^{i+2}N} = \frac{(2^i+1)}{2^{i+1}}$:

- For $i = 0, \rho = 1$, the proposed ZCZ sequence set is optimal.
- For $i > 0$, $1/2 \leq \rho < 1$ and $\lim_{i \rightarrow \infty}(\rho) = 1/2$.

6. Conclusion

In this paper, novel technique for constructing polyphase ZCZ sequence sets have been proposed. The proposed polyphase ZCZ sequence sets can be generated from a DFT matrix and their construction is more flexible than other polyphase ZCZ constructions. This construction ZCZ sequence sets is optimal or approach optimal polyphase ZCZ sequences.

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